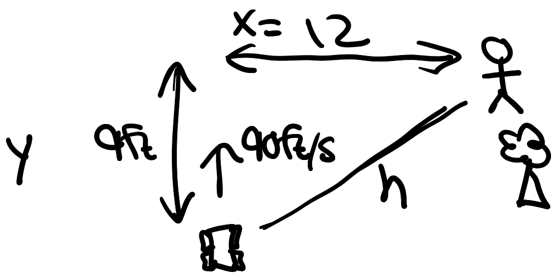
Example 17.4.1 You're cruising at ~~60~~ 90ft/s miles per hour, on a road where the speed limit is 40 miles per hour. 9 feet ahead of you behind a tree 12 feet off the road, a cop stands with a radar gun and clocks your speed. Will you get pulled over?



Know $\frac{dy}{dt}$ WTF $\frac{dh}{dt}$

Pythagorean

$$h^2 = 12^2 + y^2$$

$$\frac{d}{dt}(h^2) = \frac{d}{dt}(12^2 + y^2)$$

$$2h\frac{dh}{dt} = 0 + 2y\frac{dy}{dt}$$

$$\boxed{\frac{dh}{dt} = \boxed{\frac{2y\frac{dy}{dt}}{2h}} = \boxed{\frac{y}{h}\frac{dy}{dt}}}$$

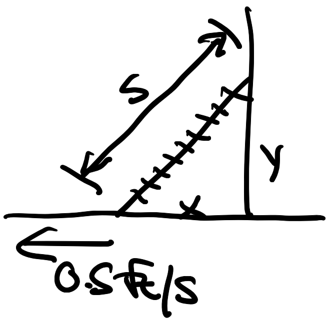
$$h^2 = 12^2 + 9^2 \iff h = 15$$

$$\frac{dh}{dt} = \frac{9}{15}90 = 54\text{ft/s}$$

$$\approx 36\text{mph}$$

Example 17.4.3 A 5 foot ladder leans against the side of a building. You grab the base of the ladder and begin sliding it away from the wall at a constant rate of 0.5 feet per second.

(a) How fast is the ladder sliding down when the base of the ladder is 3 feet away from the wall?



want to know $\frac{dy}{dt}$

we have $\frac{dx}{dt}$

$$\frac{d}{dt}\left(x^2+y^2=s^2\right)$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=-\frac{2x}{2y}\frac{dx}{dt}=-\frac{x}{y}\frac{dx}{dt}$$

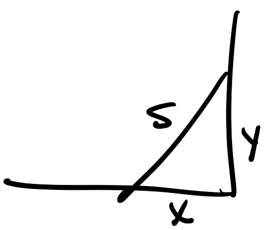
know $x=3$

$3^2+y^2=5^2$

$y=4$

$$=-\frac{3}{4}(0.5)=-0.375$$

(b) What happens to the rate of change of the height of the ladder as the base gets farther away?



$$\frac{dy}{dt}=-\frac{x}{y}\frac{dx}{dt}$$

$x \to 5$, $y \to 0$

$$\lim_{t\to\infty}\frac{dy}{dt}=\cancel{\infty}\ -\infty$$

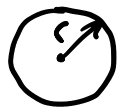
$V = \frac{4}{3}\pi r^3$

spherical

cubic

Example 17.4.4 You are blowing up a balloon at a constant rate of 3 ~~square~~ inches per second, how fast is the radius of the balloon changing when the radius of the balloon is 2 inches?

(a) Draw a diagram, and determine what rates you are trying to relate.
What are your knowns and unknowns?



Know $\frac{dV}{dt}$, want $\frac{dr}{dt}$

$\frac{dV}{dt} = 3$

$\frac{dr}{dt} = ?$

$r = 2$

(b) What equation relates the two rates?

$$V = \frac{4}{3}\pi r^3$$

(c) Take a derivative of your equation from (b).

$$\frac{d}{dt}\left((r(t))^3\right) = 3r(t)r'(t)$$

$$\frac{dV}{dt} = \cancel{\frac{4}{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2}$$

(d) Using your knowns and unknowns, how fast is the radius changing when $r = 2$?

$$\frac{dV}{dt} = \frac{3}{4(4)\pi} = \frac{3}{16\pi}$$

(e) What happens to the rate of change of the radius as more air is blown into the balloon?

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2} = \frac{3}{4\pi r^2}$$

Example 17.4.5 A water bottle is made of a cylinder of roughly 4 inches in diameter and six inches high, capped with the top half of a sphere of radius 2. Water is flowing from a faucet into the bottle at a rate of 2 cubic inches per second. $\frac{dV}{dt}$

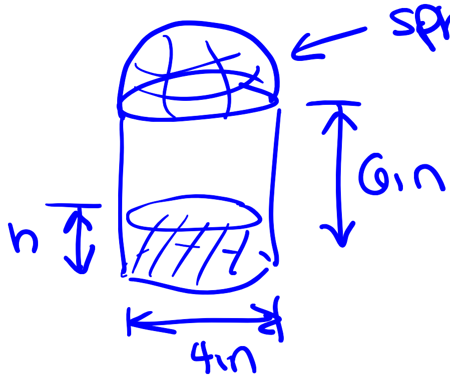
(a) When the water in the bottle is between 0 and 6 inches of height (when it's in the cylinder), show that ~~the height~~ of water is changing at a constant rate. $\frac{dh}{dt}$

(b) From 6 to 8 inches of height, the volume of the water in the bottle is

$$V = \pi\Big(48 + 4y - \frac{y^3}{3}\Big)$$

where $y+6$ is the height of the water in the bottle (we'll use y to make our calculations easier). Show that the rate of change of the water increases as the height increases.

sphere radius 2



want $\frac{dh}{dt}$, know $\frac{dV}{dt}$

$\Rightarrow$ relate V to h

$V = \pi r^2 h$

$\boxed{V = 4\pi h}$

$2 = \frac{dV}{dt} = 4\pi \frac{dh}{dt}$

trying to show $\frac{dh}{dt}$ not constant

$\frac{dh}{dt} = \frac{1}{2\pi}$

b) WTS in top, that $\frac{dh}{dt}$ is increasing

$V = \pi\left(48 + 4y - \frac{y^3}{3}\right)$

$\frac{dV}{dt} = \pi\left(4\frac{dy}{dt} - \frac{3y^2}{3}\frac{dy}{dt}\right) = \frac{dy}{dt}\pi(4-y^2)$

$2 = \frac{dy}{dt}\pi(4-y^2) \Leftrightarrow \frac{dy}{dt} = \frac{2}{\pi(4-y^2)}$ height

denominator goes to zero

$\frac{dy}{dt} \to \infty$