24.2 The Average Value of a Function: An Application of the Definite Integral

Goals

- find the average value of a function
- ~~mean value theorem for integrals~~

Question 24.2.1. How do we measure average velocity on an interval?

average velocity = $\dfrac{\text{change in pos}}{\text{change in time}}$

(rate of change of pos wrt time)

Suppose $F(x)$ is a fn st. $F'(x) = f(x)$
(F is an antideriv)

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_1^3 (t+1)\,dt = \frac{1}{2}t^2 + t\Big]_1^3 = \left(\frac{1}{2}(3)^2 + 3\right) - \left(\frac{1}{2}(1)^2 + 1\right)$$

$\frac{33}{2} - \frac{3}{2}$

$= 15$

$\frac{d}{dt}\frac{1}{2}t^2 = \frac{2}{2}t = t$

$\frac{d}{dt}1 = 0$

but antideriv of 1 is $t + C$

$- = 1$ $-$

Definition 24.2.2. The average velocity of an object is

$$v_{\text{avg}} = \frac{\text{(net change in position)}}{\text{(change in time)}}$$

displacement
(how far moved)

Example 24.2.3. An object falls off of the roof of a building. At time $t = 1$ the object has fallen 5 meters. At time $t = 4$ seconds, the object has fallen 80 meters. What is the average velocity of the object from $t = 1$ to $t = 4$ seconds?

$$V_{avg} = \frac{(\text{net change in pos})}{(\text{change in time})} = \frac{80-5}{4-1} = \frac{75}{3} = 25$$

more like 9.8t

Example 24.2.4. An object falls off of the roof of a building. The object's velocity can be modeled using the equation

$$\boxed{v(t) = 10t}$$

t is seconds
v(t) in m/s

What is the average velocity of the object from $t = 1$ to $t = 4$ seconds?

$$V_{avg} = \frac{(\text{net change in pos})}{\text{change in time}} = \frac{(\text{net change in pos})}{4-1}$$

WARNING: it's not

net change in velocity

$$\frac{v(4)-v(1)}{4-1} = \frac{m/s}{s} = m/s^2$$

if $v(t) = 10t$

$$(\text{net change in velocity}) = \int_a^b v(t)\,dt$$

so

$$\text{net change in pos} = \int_1^4 10t\,dt$$

$$= 5t^2\Big]_1^4 = 5(4)^2 - 5(1)^2$$

$$= 80 - 5 = 75$$

$$V_{avg} = \frac{\text{change in pos}}{\text{change in time}} = \boxed{\frac{\int_1^4 10t\,dt}{4-1}} = \frac{75}{3} = 25$$

in general

if $s(t)$ is pos
$s'(t) = v(t)$

$$V_{avg} = \underbrace{\frac{1}{b-a}}_{\text{change in time}} \underbrace{\int_a^b v(t)\,dt}_{\text{change in pos}} = \frac{s(b)-s(a)}{b-a}$$

Theorem 24.2.5. If the velocity of an object is $v(t)$, then the average velocity from $t = a$ to $t = b$ is

change in time

$$v_{\text{avg}} = \frac{1}{b-a}\int_a^b v(t)\,dt$$

change in pos.

- we can do this with any rate fn
  for ex. $r(t)$ measures rate of change of water flowing over a dam
  what is average rate of change?
  $$\frac{1}{b-a}\int_a^b r(t)\,dt$$
  change in time
  net change
- in general actually works for any fn
  average value of $f(x)$ on $[a,b]$ is
  $$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Example 24.2.6. Suppose the price demand function for a commodity is

$$x = 1000 - 50p,$$

eg. $x(10) = 500$ means when $p = \$10$ 500 want the item

where x is the number of people that will buy the item at the price p in dollars. On average, how many people will buy the item if the price is set between 14 and 16 dollars?

NOT $\dfrac{x(16)+x(14)}{16-14}$

~~aside~~ remark
test scores
75, 78, 93
whats average?
$\dfrac{75+78+93}{3}$
is it $\dfrac{75+93}{2}$? No!

average value of $f(x)$ on $[a,b]$ is

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$x_{avg} = \int_{14}^{16}(1000-50p)\,dp = 1000p - 25p^2\Big]_{14}^{16}$$

$$= \left(1000(16)-25(16)^2\right) - \left(1000(14)-25(14)^2\right)$$

$$= 500 \text{ people}$$

Interpretation:
On average if price is b/w \$14 and \$16 expect 500 ppl to want item.

Theorem 24.2.7. The average value of a function $f(x)$ on $[a, b]$ is

$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

Example 24.2.8. Find the average value of each of the following functions on the interval $[1, 3]$.

(a) $f(x) = 3$

(b) $g(x) = x + 1$

(c) $h(x) = -3x^2 + 8x$