Yale University
Department of Mathematics

MATH 118 — Exam 1 Practice Problems — SPRING 2020

NAME: ______________________________

Instructor Name: ______________________________ Section: __________

This ***is not*** an exhaustive list of problems you should expect to see on the actual exam. It is not representative of the length of the in class exam, and problems on the actual exam may be different than those seen here. Look at the homework, quizzes, in class notes, or extra suggested problems from the book for more practice.

The following message appears on the actual exam:

Read This First!

- Please read each question carefully. Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- No calculators permitted.

1. If the statement is always true, circle True. If the statement is sometimes false, circle False. In each case, write a careful and clear **justification** or **counterexample**.

   (a) If A and B are $n \times n$ matrices and AB is invertible, then B is invertible. True False

   (b) Let A be an $m \times n$ matrix where $m > n$. If A has n pivots, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b} \in \mathbb{R}^m$. True False

2. Which of the following correctly describes the system associated to the augmented matrix:

$$\left[\begin{array}{cc|c} 2 & -8 & 0 \\ -1 & 4 & 0 \end{array}\right]$$

   (a) Inconsistent
   (b) Consistent with exactly one solution which is nontrivial
   (c) Consistent with exactly one solution which is trivial
   (d) Cosistent with infinitely many solutions, all of which are nontrivial
   (e) Consistent with infinitely many solutions, one of which is trivial

3. Which of the following sets are linearly independent?

   (a) $\left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

   (d) $\left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \end{bmatrix} \right\}$

4. For which matrix A below, do the columns of A span $\mathbb{R}^3$?

   (a) $A = \begin{bmatrix} 2 & 1 & 7 & 0 \\ 0 & -1 & 3 & -4 \\ 0 & 0 & 3 & 5 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & -4 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 9 & -1 & -5 \\ 0 & -1 & 3 \end{bmatrix}$

   (d) $A = \begin{bmatrix} -5 & 1 & 2 \\ 0 & 4 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

5. The following matrix A is in what form?

$$A = \begin{bmatrix} 1 & 4 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) Echelon form
   (b) Reduced row echelon form (RREF)
   (c) Neither

6. Given the linear system below, write the solution set in parametric vector form.

$$\begin{cases} 2x_1 - 6x^2 + 4x_3 - 6x_4 = -18 \\ 3x_1 \qquad\quad - 2x_3 \qquad\quad = 3 \end{cases}$$

7. Consider the following system of equations where $a \neq 0$. Prove that if $a \neq 24$ then the system has exactly one solution.

$$\begin{cases} ax - 6y = -1 \\ 4x - y = 3 \end{cases}$$

8. Let $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 4 & 0 \\ 3 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -4 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 \\ -2 & 6 \end{bmatrix}$.

   Compute A^{-1}, B^{-1} and C^{-1} if they exist.

9. Let $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 4 & 0 \\ 3 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -4 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 \\ -1 & 6 \end{bmatrix}$.

   Compute the following. If any operations are not possible, explain why not.

   (a) $B^T A$

   (b) ABC.

10. Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ k \\ 1 \end{bmatrix} \right\}$.

    (a) For what value(s) of k is the set S linearly independent? (Show how you arrive at your answer)

    (b) Let $k = 2$. Is the vector $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ in Span$S$? Explain why or why not.

11. For each of the following, determine whether the matrix described is invertible or not invertible, and **give a reason** for each one.

    (a) A is a 3×3 matrix, and the solution set to $A\mathbf{x} = \mathbf{0}$ is a plane.

    (b) B is a square matrix, and there is a matrix D such that $BD = I$.

    (c) C is an 3×3 matrix and the columns of C span all of $\mathbb{R}^3$.

12. Determine the value(s) of h such that the augmented matrix yields a consistent linear system

$$\left[\begin{array}{cc|c} 1 & 4 & -2 \\ 3 & h & -6 \end{array}\right]$$

13. Find an equation involving g, h, k that makes the following correspond to a consistent system

$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right]$$

14. Determine if $\begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$

15. Show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in Span $\left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ for all h and k.

16. Determine if the system has a nontrivial solution

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 0 \\ -2x_1 - 3x_2 - 4x_3 &= 0 \\ 2x_1 - 4x_2 + 9x_3 &= 0 \end{aligned}$$

17. Determine if the vectors are linearly independent.

$$\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$$

18. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$,
$D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$ and $E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$. Compute $A + 3B$, $2C - 3E$, DB and EC.

19. Find the inverse of $A = \begin{bmatrix} 7 & 3 \\ -6 & -3 \end{bmatrix}$ and use it to solve the matrix equation $A\mathbf{x} = \begin{bmatrix} -9 \\ 4 \end{bmatrix}$

20. Let V be the first quadrant of the xy-plane.
    (a) If $\mathbf{u}$ and $\mathbf{v}$ are in V, is $\mathbf{u} + \mathbf{v}$?
    (b) Find a specific vector $\mathbf{v}$ in V and a scalar c such that $c\mathbf{v}$ is not in V.

21. Let H be the set of points inside and on the unit circle in the xy-plane. Find a specific example that shows that H is not a subspace of $\mathbb{R}^2$.

22. Find a basis for the null space and column space of the matrix

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 5 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & -8 & 0 & 16 \end{bmatrix}$$

23. Find a unit vector that's orthogonal to $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

24. Show that $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is orthogonal to Span $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \right\}$

25. Find a vector that's orthogonal to Span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \right\}$