MATH 118
Midterm Test #1 Problems

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1. Let $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -2 \\ -4 \\ 1 \end{bmatrix}$.

   (a) Calculate $3\mathbf{u} + \mathbf{v} - 2\mathbf{w}$.

   (b) Calculate $\|\mathbf{u}\|$, $\|\mathbf{v}\|$ and $\|\mathbf{w}\|$.

   (c) Is $\mathbf{u}$ perpendicular to $\mathbf{v}$ or $\mathbf{w}$? Is $\mathbf{v}$ perpendicular to $\mathbf{w}$? Are any of the vectors parallel?

   (d) Find the angle between $\mathbf{v}$ and $\mathbf{w}$.

   (e) Are the vectors $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ linearly independent?

2. Let

$$A = \begin{bmatrix} 0 & -2 & 1 & 3 \\ 2 & 5 & -1 & -1 \\ 1 & 4 & 0 & 7 \end{bmatrix}, \; B = \begin{bmatrix} 3 & 4 \\ 0 & -2 \\ 1 & -1 \\ -8 & 2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 0 & 3 \\ -5 & 2 \end{bmatrix}.$$

   Calculate each of the following, or explain why the operation doesn't make sense.

   (a) AB
   (b) BA
   (c) AC
   (d) BC
   (e) CB
   (f) ABC
   (g) A^T
   (h) A^{-1}
   (i) C^{-1}

3. One serving of Post Shredded Wheat contains 160 calories, 5g of protein, 6g of fiber, and 1g of fat. One serving of Crispix contains 110 calories, 2g of protein, 0.1g of fiber, and 0.4g of fat.

   Set up a matrix B and a vector $\mathbf{u}$ such that $B\mathbf{u}$ gives the amounts of calories, protein, fiber, and fat contained in a mixture of three servings of Shredded Wheat and two servings of Crispix.

4. Find all of the solutions to the following system of equations, or show that no solution exists.

$$\begin{cases} 3x + 2z = -5 \\ 2x - y + z = 4 \\ x + y - z = -2 \end{cases}$$

5. Find all of the solutions to the following system of equations, or show that no solution exists.

$$\begin{cases} w + x - 2y + 3z = 4 \\ 2w + 3x + 3y - z = 3 \\ 5w + 7x + 4y + z = 5 \end{cases}$$

6. Find all of the solutions to the following system of equations, or show that no solution exists.

$$\begin{cases} w + x - 2y + 4z = 5 \\ 2w + 2x - 3y + z = 3 \\ 3w + 3x - 4y - 2z = 1 \end{cases}$$

7. Let

$$A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 3 & 1 & 1 \\ -1 & 2 & 1 & 0 \\ 2 & -3 & 0 & 4 \end{bmatrix}.$$

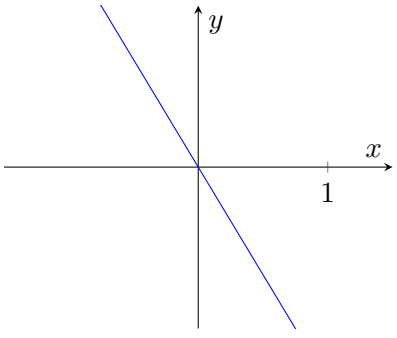
Find A^{-1}, or show that A is not invertible.

10. Which of the following sets, described either using set notation or illustrated in blue, form subspaces of either $\mathbb{R}^2$ or $\mathbb{R}^3$? For those that aren't, explain which property of subspaces is violated.

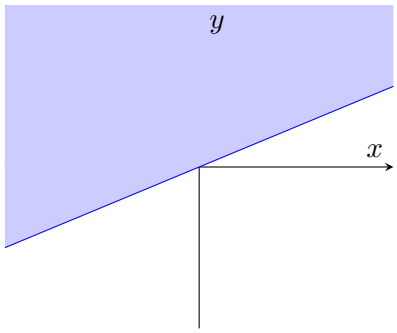
(a) $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; y = 1 - x \right\}$

(b) $\left\{ c \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + d \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \;\middle|\; c, d \text{ are in } \mathbb{R} \right\}$

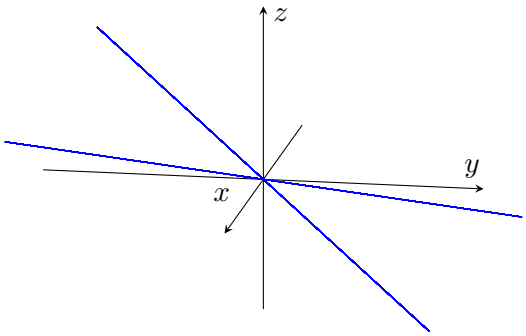
(c)



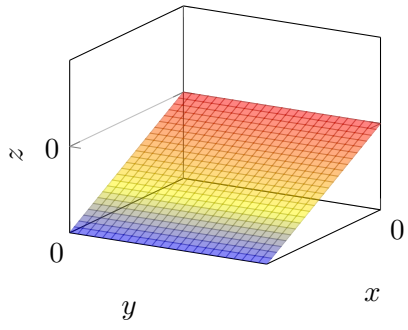
(d)



(e)



(f)



11. Let

$$A = \begin{bmatrix} 2 & -6 \\ -1 & 3 \\ -4 & 12 \\ 3 & -9 \end{bmatrix}.$$

Find a basis for Null(A).

12. Let

$$A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix}.$$

(a) Find p and q such that Null(A) is a subspace of $\mathbb{R}^p$ and Col(A) is a subspace of $\mathbb{R}^q$.

(b) Find a basis for Col(A).

13. Let U be the collection of all vectors of the form $\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix}$. Find vectors $\mathbf{u}$ and $\mathbf{v}$ such that $U = \text{span}\{\mathbf{u}, \mathbf{v}\}$. Is U a subspace of $\mathbb{R}^4$?

14. Find the value(s) of h for which the vectors

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$$

are linearly dependent.

15. Determine whether the following statements are true or false. If true, provide justification. If false, provide a counterexample.

(a) If A and B are square matrices (*i.e.* the number or rows is equal to the number of columns), then $AB = BA$.

(b) If $AB = 0$, then either $A = 0$ or $B = 0$.

(c) If $\mathbf{u}$ and $\mathbf{v}$ are in the null space of A, then $3\mathbf{u} - 10\mathbf{v}$ is also in the null space of A.

(d) If A is a square matrix, then A is invertible.