MATH 118

Written Assignment #6

For the written homework assignment, you are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence of your solutions. Your solutions must have your name written on the top of the first page.

Your written assignment should be submitted as a pdf electronically through Canvas. Please email Pam if you are experiencing any technical difficulties.

1. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix}$. Calculate each of the following.
   (a) $\mathbf{u} \times \mathbf{v}$
   (b) $\mathbf{u} \times \mathbf{w}$
   (c) $\mathbf{v} \times \mathbf{w}$
   (d) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$
   (e) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$
   (f) $\mathbf{v} \times \mathbf{u}$ (Hint: Use your answer from (a).)

2. Consider the line L through $(-4, -6, 1)$ and $(-2, 0, -3)$.
   (a) Find a vector equation for L.
   (b) Is L parallel to the line through $(10, 18, 4)$ and $(5, 3, 14)$?

3. Find the equation of the plane passing through the points $(0, 1, 1)$, $(1, 0, 1)$ and $(1, 1, 0)$.

4. Find the equation of the plane that passes through the point $(3, 5, -1)$ and contains the line with vector equation $\mathbf{r}(t) = \begin{bmatrix} 4-t \\ 2t-1 \\ -3t \end{bmatrix}$.

5. A tire company produces two different types of car tires: winter tires and all-season tires. It costs the company \$15 to make each winter tire and \$10 to make each all-season tire. In addition, as the company is small, the budget for its manufacturing costs is \$50,000 (*i.e.* the total manufacturing costs cannot exceed \$50,000).
   (a) Write down a function $C(x, y)$ representing the manufacturing cost of producing x winter tires and y all-season tires.
   (b) Find $C(1000, 2500)$ and write a sentence interpreting its meaning.

6. In 1928, Charles Cobbs and Paul Douglas published a study in which they modeled the growth of the American economy during 1899-1922. Their model related the total production of the economy P (*i.e.* the monetary value of all goods produced in a year) to the amount of labour L (*i.e.* the total number of person-hours worked in a year) and capital K (*i.e.* the monetary value of all machinery, equipment and buildings) invested in the economy. In particular, the model they used was of the form

$$P = P(L, K) = bL^{\alpha}K^{1-\alpha}.$$

This is known as the *general Cobbs-Douglas production function.*

(a) Show that, by taking the natural logarithm of both sides, the general Cobbs-Douglas production function above can be expressed as

$$\ln\left(\frac{P}{K}\right) = \ln(b) + \alpha \ln\left(\frac{L}{K}\right).$$

(b) Notice that the equation in (b) is linear in α and $\ln(b)$. Use the equation from (b) and the data below to generate a system of linear equations in α and $\ln(b)$.

Year	P	L	K
1899	100	100	100
1904	122	121	138
1910	159	147	208
1916	225	183	298
1922	240	161	431

(c) Find the least-squares solution to the system of equations from (c) (round to the nearest thousandth, *i.e.* 2/3 would be rounded to 0.667). **You may use a graphing calculator or computer to do the row-reduction.**

(d) Use your answer to (d) to find the "best" Cobbs-Douglas model for the production of the economy during 1899-1922 (*i.e.* find the Cobbs-Douglas function $P(L, K)$ corresponding to the least-squares solution from (d)).[1]

[1] Cobbs and Douglas built their model by doing exactly this – they just used more data to construct their system of equations before finding the least-squares solution.