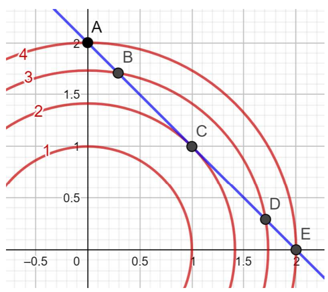
Lecture 10 Global Extrema via /Lagrange Multipliers

Stewart 14.1, McCallum 12.3, 12.5

- understand and use Lagrange multipliers to answer questions about global extrema

Example 10.1. Use contour map of $f(x, y) = x^2 + y^2$ and the graph of the curve $x + y = 2$ to find the absolute minimum of the function $f(x, y)$ subject to the constraint $x + y = 2$ (that is, the absolute minimum of f on the boundary). (find a graph at `https://www.geogebra.org/m/pvj8wwrs`)



Question 10.2. Can we make a general statement about the gradient of a function and the gradient of a constraint condition at a point of local extrema on the boundary?

> **Theorem 10.3.** (Lagrange Multipliers) Let $f(x, y)$ be a differentiable function defined over a region R, with the boundary of R given by a differentiable function $g(x, y) = c$ (where c is any constant). If $f(x, y)$ has a local extremum on the boundary at $P(a, b)$, and $\nabla g(x, y) \neq 0$, then
>
> $$\nabla f(a, b) = \lambda \nabla g(a, b),$$
>
> for some constant λ. Here, $g = c$ is called a **constraint curve** and λ is called the **Lagrange Multiplier**.

Example 10.4. Use Lagrange multipliers to find the critical points of $f(x, y) = x^2 + y^2$ subject to $x + y = 2$.
(find a graph at `https://www.geogebra.org/m/zmf4fwfn`)

Example 10.5. Find the global maximum and global minimum of the function $f(x, y, z) = x + z^2$ on the sphere $x^2 + y^2 + z^2 = 1$.

Example 10.6. Find global extrema of $f(x, y) = x + 2y$ on the disk $x^2 + y^2 \leq 5$. (find a graph at `https://www.geogebra.org/m/nesu9yym`)

Example 10.7. Find the point on $x + y + z = 1$ closest to the origin. (find a graph at `https://www.geogebra.org/m/m3jzzek4`)