3 Vector valued functions

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3. Key Ideas So far, the functions that we've studied in calculus have been real-valued, taking values in $\mathbb{R}$ and outputting values in $\mathbb{R}$. In this chapter, we will study functions whose outputs are vectors, primarily in three dimensions.

- define and understand vector-valued functions
- differentiate vector-valued functions
- understand what the derivative represents geometrically

3.1 Vector valued functions

Definition 3.1. A vector-valued function is a function whose input is a number and output is a vector.

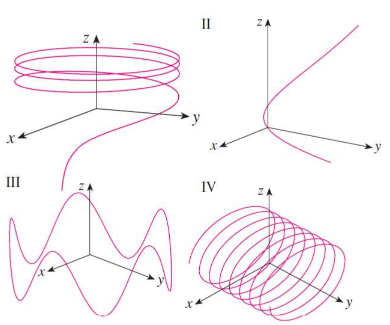
Example 3.2. What do the curves
$\mathbf{r}(t) = \langle \cos(t), \sin(t), 0 \rangle$ and $\mathbf{s}(t) = \langle \cos(t), \sin(t), t \rangle$ look like?

Example 3.3. What does the curve

$$x = \cos t,\ y = \sin t,\ z = 2 - \sin t \text{ for } 0 \le t \le 2\pi$$

look like?

Example 3.4. Match the graphs to their equations:



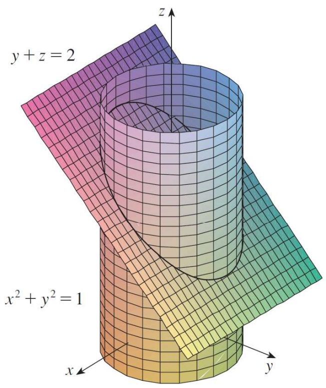
(a) $\mathbf{r}(t) = \langle \cos 4t, t, \sin 4t \rangle$

(b) $\mathbf{r}(t) = \langle \cos t, \sin t, \sin 5t \rangle$

(c) $\mathbf{r}(t) = \langle \cos t, \sin t, \ln t \rangle$

(d) $\mathbf{r}(t) = \langle t, t^2, e^{-t} \rangle$

Example 3.5. Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.



3.2 Derivatives

Definition 3.6. Given a vector valued function $\mathbf{r}(t) = \langle r_1(t), r_2(t), r_3(t)\rangle$, the derivative $\mathbf{r}'(t)$ is given by

$$\mathbf{r}'(t) = \langle r_1'(t), r_2'(t), r_3'(t)\rangle$$

Example 3.7. If $\mathbf{r}(t) = \langle 2\cos t, \sin t, t\rangle$, find and interpret $\mathbf{r}'(t)$.

Example 3.8. Find the equation of the tangent line to $\mathbf{r}(t) = \langle 2t^2, t+1, -t \rangle$ at the point $(8, 3, -2)$.

Example 3.9. Find the point(s) at which $\langle \cos t, \sin t, 2 - \sin t \rangle$ for $0 \leq t \leq 2\pi$ achieves a maximum.