MATH 118
Vector Valued Functions, Derivatives

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1. A vector function is a function that takes an input $t \in \mathbb{R}$ and outputs a vector $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$. The real-valued functions $x(t)$, $y(t)$ and $z(t)$ are called parameterizations. Find vector functions and parameterizations for the following curves.

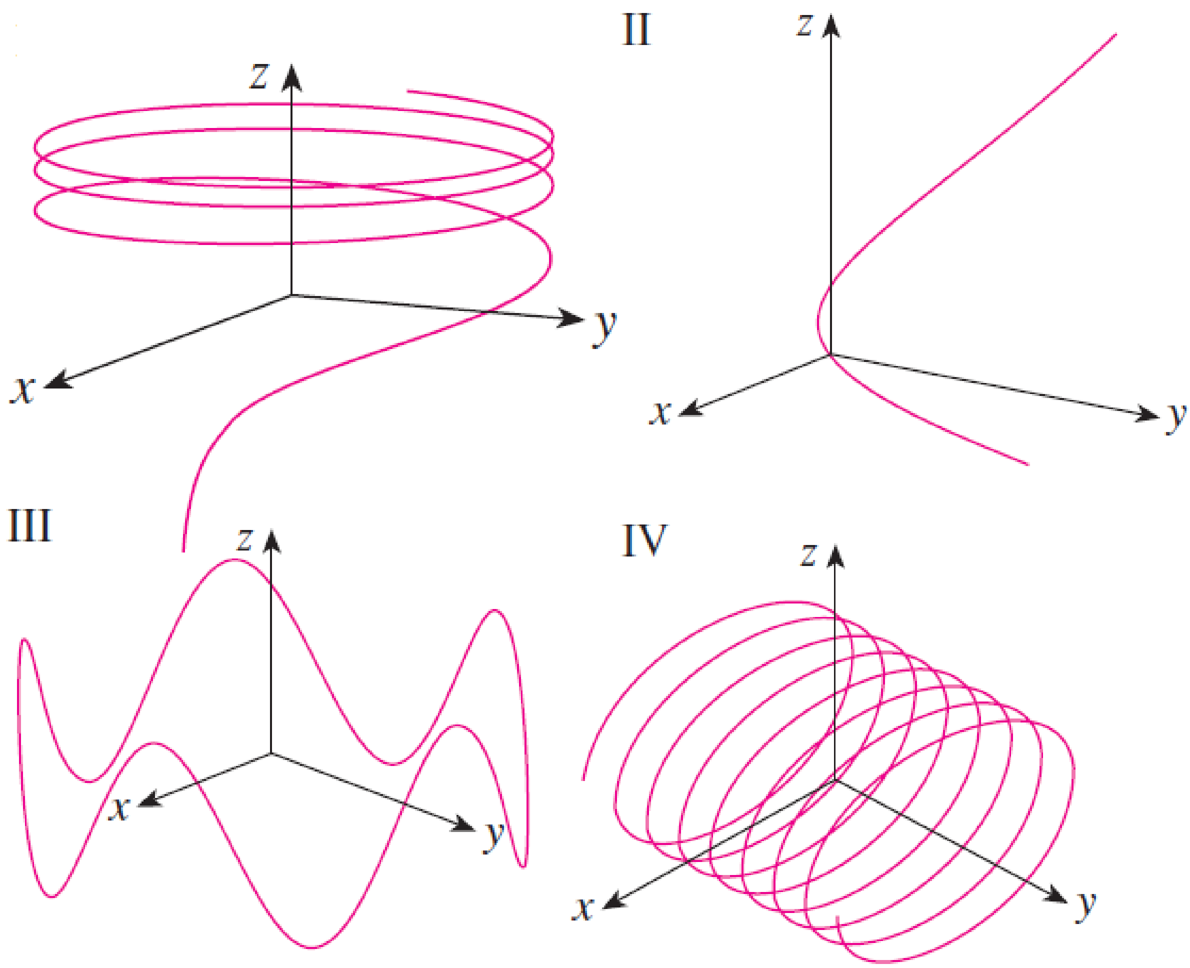
   (a) The line segment connecting $P(1, -1, 4)$ to $Q(4, 7, 1)$

   (b) The line segment connecting $P(0, -1, 3)$ to $Q(\frac{1}{2}, \frac{1}{3}, \frac{1}{4})$

   (c) The arc of radius 3 from $(3, 0)$ to $(0, 3)$

   (d) The triangle with vertices $(0, 0)$, $(5, 0)$, and $(0, 5)$

2. Match the graphs to their equations:



(a) $\mathbf{r}(t) = \langle \cos 4t, t, \sin 4t \rangle$

(b) $\mathbf{r}(t) = \langle \cos t, \sin t, \sin 5t \rangle$

(c) $\mathbf{r}(t) = \langle \cos t, \sin t, \ln t \rangle$

(d) $\mathbf{r}(t) = \langle t, t^2, e^{-t} \rangle$

3. What are the derivatives of the following functions?

(a) x^n
(b) e^x
(c) b^x
(d) $\ln x$
(e) $\log_b(x)$
(f) $\sin(x)$
(g) $\cos(x)$
(h) $f(x)g(x)$
(i) $f(x)/g(x)$
(j) $f(g(x))$

4. Find derivatives of the following functions.

(a) $e^x \sin x$

(b) $\dfrac{x^3}{\ln x}$

(c) e^{x^2}

5. A store has been selling 200 flat-screen TVs a week at \$350 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of TVs sold will increase by 20 a week. Find the demand function and the revenue function. How large a rebate should the store offer to maximize its revenue?

6. Find a vector valued function that represents the curve of intersection of the cylinder $x^2+y^2 = 4$ with the plane $3y + z = 7$. What is the maximum height of this curve?

7. When two curves intersect at a point P, their angle of intersection is the angle between their tangent vectors at P. Find the point at which $\mathbf{r}_1(t) = \langle t, 3 - t, 35 + t^2\rangle$ and $\mathbf{r}_2(t) = \langle 7 - t, t - 4, t^2\rangle$ intersect, and their angle of intersection.

8. When the vector function $\mathbf{s}(t)$ represents position, the first derivative $\mathbf{s}'(t) = \mathbf{v}(t)$ represents velocity and the second derivative $\mathbf{s}''(t) = \mathbf{a}(t)$ represents acceleration. Suppose the position (x, y) of a soccer ball, kicked off of Kline Biology Tower, is modeled by the equation

$$\mathbf{r}(t) = \langle 25t, 270 + 50t - 5t^2 \rangle$$

where t is in seconds, and (x, y) is the position of the ball relative to the base of the tower.

(a) graph the position of the ball

(b) how fast is the ball going horizontally at time t?

(c) how fast is the ball going vertically at time t?

(d) how fast is the ball going overall at time t?

(e) how high does the ball get?

(f) when does the ball hit the ground?