Lecture 4 Functions of several variables

Stewart 14.1, McCallum 12.3, 12.5

Lecture 4. Key Ideas So far, the functions that we've studied in calculus have been real-valued, taking values in $\mathbb{R}$ and outputting values in $\mathbb{R}$. In this chapter, we will study functions whose outputs are vectors, primarily in three dimensions.

- understand what a function of two variables is
- identify the domain fo a function of two variables
- find the level curves of a function of two variables
- identify graphs of paraboloids, cones, spheres, planes and cylinders

Lecture 4.1 Functions of more than one variable

Definition 4.1. A function of two variables is one whose input is two numbers and output is a single number. Similar for functions of three or more variables.

Example 4.2. Describe $f(x, y) = 2 - x + 2y$ geometrically.

Example 4.3. What does the graphs of $f(x, y) = x^2 + y^2$ look like?

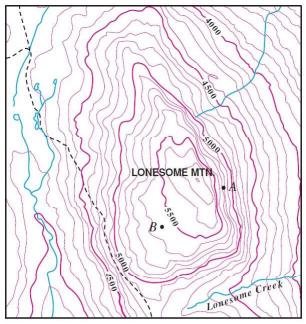
Lecture 4.2 Domain

Definition 4.4. The domain of $f(x, y)$ is the set of all (x, y) for which f is defined.

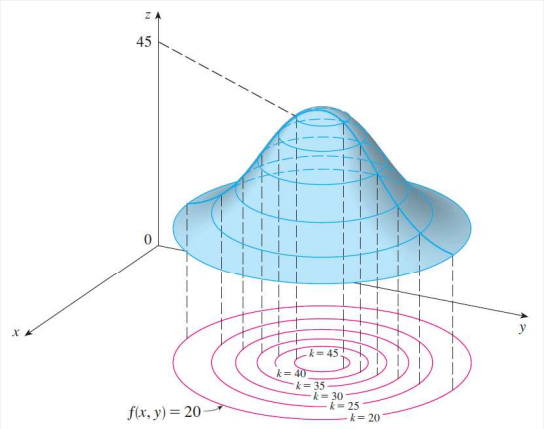
Example 4.5. Describe the function $\sqrt{1 - x^2 - y^2}$ geometrically. What is its domain?

Lecture 4.3 Level Curves and Contour Maps

Example 4.6. Below is a map of Lonesome mountain. What do the lines represent?



Definition 4.7. Given a function $z = f(x, y)$, the **level curves** are the curves obtained by setting $z = c$. A **contour map** is a drawing of several evenly spaced level curves.



Example 4.8. Sketch a contour map for $z = x^2 + y^2$.