MATH 118
Planes in $\mathbb{R}^3$ and functions of 2 variables

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1. Find a non-zero vector perpendicular to the plane $2x - y + 3z = 5$.

2. Find the equation of the plane which passes through the point $P(0, 1, 1)$ and is perpendicular to the line given by $\mathbf{r}(t) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

3. Find the equation of the plane which passes through the point $P(2, -1, 1)$ and contains the non-parallel vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$.

4. Find and sketch the domain for each of the following functions.

   (a) $f(x, y) = \dfrac{\sin(xy)}{\sqrt{x + y}}$

   (b) $g(x, y) = \ln\left(\dfrac{x}{y}\right)$

5. When x and y represent physical quantities, sometimes there are additional restrictions on the domain due to physical limitations of the quantities. In such a case we will refer to the result as the "physical domain" of the function.

   If a company sells x units of good A and y units of good B, their revenue is $R(x, y) = 40x+22y$. What is the "physical domain" of $R(x, y)$?

6. Consider the surface defined by $z = 1 - y$.

   (a) What kind of surface is defined by the equation?

   (b) Make a rough sketch of the surface.

7. Consider the surface defined by $x^2 + y^2 = 1$.

   (a) What kind of surface is defined by the equation?

   (b) Make a rough sketch of the surface.

MATH 118
Contour plots and partial derivatives

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1. For each of the following functions, make a contour plot for $f(x, y)$ and use it to help you sketch the graph of the surface $z = f(x, y)$.

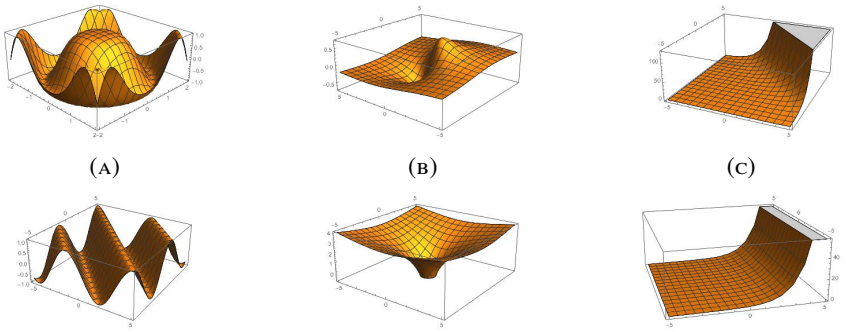
   (a) $f(x, y) = \sqrt{x^2 + y^2}$

   (b) $f(x, y) = x^2 + y^2$

2. The contour plots in question 1 are very similar. Write a sentence or two describing the difference in the corresponding surfaces.

3. Match the following equations with their graphs and contour plots.

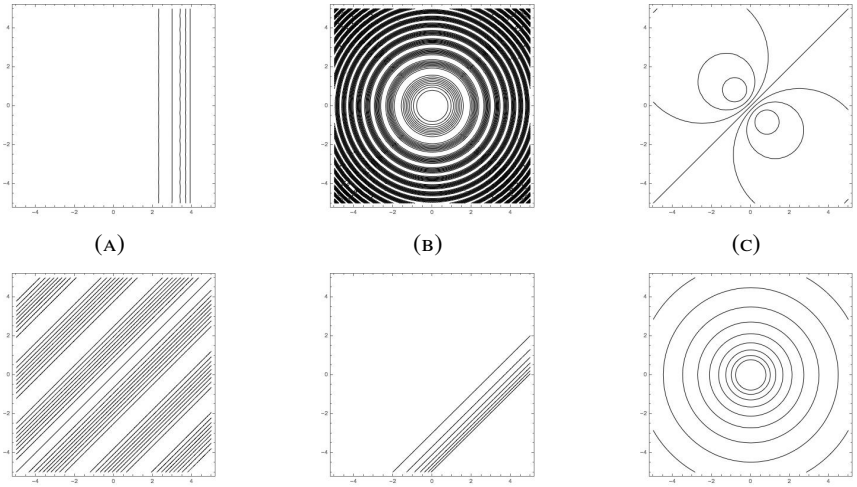
$f(x,y) = e^{x-y}$	Graph _____	Contour _____
$f(x,y) = e^{x}$	Graph _____	Contour _____
$f(x,y) = \cos(x-y)$	Graph _____	Contour _____
$f(x,y) = \ln(x^2+y^2)$	Graph _____	Contour _____
$f(x,y) = \frac{x-y}{1+x^2+y^2}$	Graph _____	Contour _____
$f(x,y) = \cos(x^2+y^2)$	Graph _____	Contour _____



(A) (B) (C)

(D) (E) (F)

FIGURE 1. The graphs for problem



(A) (B) (C)

(D) (E) (F)

FIGURE 2. The contours for problem

4. Calculate the partial derivatives of each of the following functions.

   (a) $f(x, y) = 1 + x \sin(xy)$

   (b) $g(x, y, z) = \dfrac{z}{1 + x^2} + e^{xyz}$

5. Recall your solutions to question 2.

   (a) Use your contour plots to determine the sign of $f_x(1, 1)$ and $f_y(1, 1)$ for each function.

   (b) Do your answer to (a) agree with your sketches of the surfaces $z = f(x, y)$ in each case?

   (c) Now calculate $f_x(1, 1)$ and $f_y(1, 1)$ in each case and verify that the result agrees with your answers to the questions above.