MATH 118
Local extrema

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1. Rederive the equation of the plane tangent to the surface $z = f(x, y)$ at (x_0, y_0) by completing the following steps.

   (i) Recognize the surface as the level surface of a function of three variables $F(x, y, z)$.

   (ii) Find a vector perpendicular to the surface, and therefore perpendicular to the tangent plane, at (x_0, y_0).

   (iii) Find the z-coordinate of the point on the surface corresponding to $x = x_0$ and $y = y_0$. Then write down the equation of the tangent plane.

2. Find all of the critical points of the function $f(x, y) = x^2 + 2y^2 + x^2y + 4$.

3. To better understand the Second Derivative Test, let's look at three simple cases. For each of the following functions, find and classify all of the critical points. Then make a rough sketch of the graph of the function.

   (I) $f(x, y) = x^2 + y^2$

   (II) $f(x, y) = 1 - x^2 - y^2$

   (III) $f(x, y) = y^2 - x^2$

4. For each of the critical points found in question 2, classify it as either a local minimum, local maximum or a saddle point.

5. Find and classify all of the critical points of $f(x, y) = (x - 2y)(4 - xy)$.